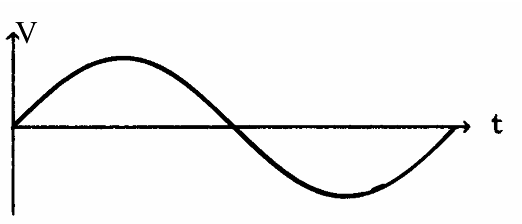
Root-Mean-Square value

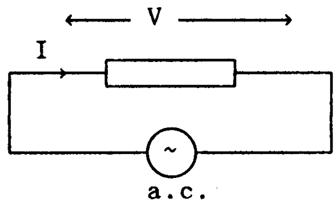
When a time-varying voltage V(t) is applied to a resistor, the power dissipated in it, V^2/R is also time-dependent. The average power is the energy consumed in a cycle over the time of one period. **If we were to use a steady d.c. voltage to give the same average power, then this "d.c. equivalent steady voltage" is called the root mean square voltage of the a.c.**

I. Complete sinusoidal waveform:

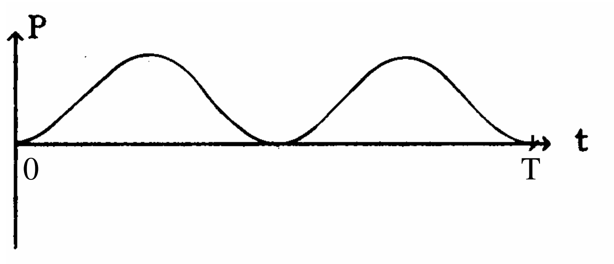
A sinusoidal a.c. is applied to a resistor

$V = V_0 \sin \omega t$





The instantaneous power dissipated in R is $P = \frac{V^2}{R} = \frac{V_0{}^2}{R}\sin^2\omega t$



The average power ,

$$\overline{P} = \frac{1}{T}\int_0^T P\,dt = \frac{1}{T}\frac{V_0^2}{R}\int_0^T \sin^2\omega t = \frac{1}{2}\frac{V_0^2}{R} = \frac{\left(\frac{V_0}{\sqrt{2}}\right)^2}{R} \quad \text{..........(1)}$$

The power of a steady d.c. voltage V_{dc} is

$$P = \frac{V_{dc}^2}{R} \quad \text{.........(2)}$$

If (2) is exactly equal to (1), then

$$\frac{V_{dc}^2}{R} = \frac{(\frac{V_0}{\sqrt{2}})^2}{R}$$

This implies $V_{dc} = \frac{V_0}{\sqrt{2}}$

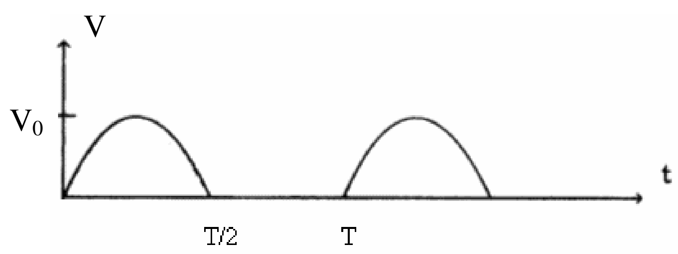
This "d.c. equivalent steady voltage" V_{dc} is called the root-mean-square voltage (V_{rms}) of the a.c. because it is calculated from the procedures "root", "mean" and "square", but in the reverse order.

1. Squaring the time-depending a.c. voltage.
2. Taking the mean of the squared V over a cycle.
3. Taking the square root of the mean.

$$V_{rms} = \sqrt{\overline{V^2}}$$

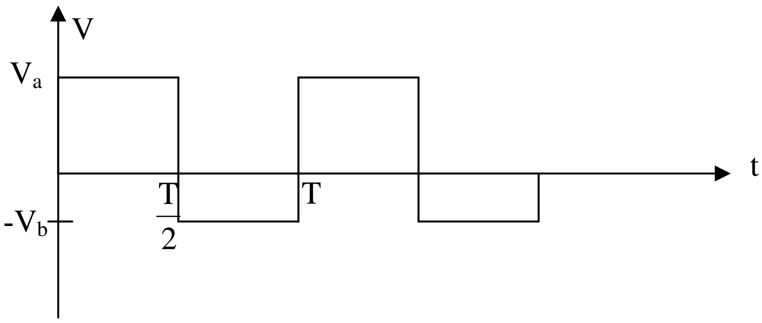
In a pure R, $\overline{P} = \frac{V_{rms}^2}{R} = I_{rms}^2 R = V_{rms} I_{rms}$

II. Half sinusoidal waveform:



$$V_{rms} = \frac{V_0}{2}$$

III. Square waveform:



$$V_{rms} = \sqrt{\frac{V_a^2 + V_b^2}{2}}$$